**EXERCISES [MAI 5.14-5.15]**

**MORE INTEGRALS - SUBSTITUTION**

**SOLUTIONS**

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## A. Paper 1 questions (SHORT)

**1.**

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| $\int\left(\cos x-\sin x+e^x+\frac{1}{x}\right)dx=\sin x+\cos x+e^x+\ln x+c$ |
| $\int\left(5\sin x-3\cos x-7e^x+\frac{1}{x}\right)dx=-5\cos x-3\sin x-7e^x+\ln x+c$ |
| $\int(x^{-3}+x^{-2}+x^{-1}+3)dx=\frac{x^{-2}}{-2}+\frac{x^{-1}}{-1}+\ln x+3x+c$ |
| $\int(4x^{-3}-12x^{-2}+6x^{-1}+3)dx=-2x^{-2}+12x^{-1}+6\ln x+3x+c$ |
| $\int(x^{\frac{1}{2}}+x^{\frac{1}{3}})dx=\frac{2}{3}x^{\frac{3}{2}}+\frac{3}{4}x^{\frac{4}{3}}+c$ |
| $\int(5x^{\frac{1}{2}}-3x^{\frac{1}{3}})dx=\frac{10}{3}x^{\frac{3}{2}}-\frac{9}{4}x^{\frac{4}{3}}+c$ |
| $\int(x^{\frac{2}{5}}+x^{\frac{3}{7}})dx=\frac{5}{7}x^{\frac{7}{5}}+\frac{3}{10}x^{\frac{10}{3}}+c$ |
| $\int(7x^{\frac{2}{5}}+10x^{\frac{3}{7}})dx=5x^{\frac{7}{5}}+3x^{\frac{10}{3}}+c$ |
| $\int\left(\cos x+\frac{1}{\cos^2 x}\right)dx=\sin x+\tan x+c$ |

**2.**

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|---|
| $\int\left(\frac{4}{x^3}-\frac{12}{x^2}+\frac{6}{x}+3\right)dx=\int(4x^{-3}-12x^{-2}+6x^{-1}+3)dx=-2x^{-2}+12x^{-1}+6\ln x+3x+c=\ldots$ |
| $\int\left(\frac{1}{4x^3}-\frac{2}{3x^2}-\frac{5}{7x}+3\right)dx=\int\left(\frac{1}{4}x^{-3}-\frac{2}{3}x^{-2}-\frac{5}{7}x^{-1}+3\right)dx=-\frac{1}{8}x^{-2}+\frac{2}{3}x^{-1}-\frac{5}{7}\ln x+3x+c=\ldots$ |
| $\int 4x^2\left(x-\frac{1}{x^3}\right)dx=\int\left(4x^3-\frac{4}{x}\right)dx=x^4-\ln x+c$ |
| $\int(5\sqrt{x}-3\sqrt[3]{x})dx=\int(5x^{\frac{1}{2}}-3x^{\frac{1}{3}})dx=5\frac{2}{3}x^{\frac{3}{2}}-3\frac{3}{4}x^{\frac{4}{3}}+c=\frac{10}{3}x^{\frac{3}{2}}-\frac{9}{4}x^{\frac{4}{3}}+c$ |
| $\int(5\sqrt{x^3}-3\sqrt[3]{x^2})dx=\int(5x^{\frac{3}{2}}-3x^{\frac{2}{3}})dx=2x^{\frac{5}{2}}-\frac{9}{5}x^{\frac{5}{3}}+c$ |
| $\int 5x(\sqrt{x}+x^3)dx=\int(5x^{\frac{3}{2}}+5x^4)dx=2x^{\frac{5}{2}}+x^5+c$ |
| $\int\frac{1+x}{x}dx=\int\left(\frac{1}{x}+1\right)dx=\ln x+x+c$ |
| $\int\frac{2x^7+5x+4}{3x^2}dx=\int\left(\frac{2}{3}x^5+\frac{5}{3}x^{-1}+\frac{4}{3}x^{-2}\right)dx=\frac{2}{3}\frac{x^6}{6}+\frac{5}{3}\ln x+\frac{4}{3}\frac{x^{-1}}{-1}+c=\frac{x^6}{9}+\frac{5}{3}\ln x-\frac{4}{3}x^{-1}+c$ |
| $\int\frac{2x\cos^2 x+1}{\cos^2 x}dx=\int\left(2x+\frac{1}{\cos^2 x}\right)dx=x^2+\tan x+c$ |

**3.**

| | |
|---|---|
| $\int \sin(5x+3)dx$ | $-\frac{\cos(5x+3)}{5}+c$ |
| $\int \cos(5x+3)dx$ | $\frac{\sin(5x+3)}{5}+c$ |
| $\int e^{5x+3}dx$ | $\frac{e^{5x+3}}{5}+c$ |
| $\int (5x+3)^3 dx$ | $\frac{(5x+3)^4}{20}+c$ |
| $\int \sin 5x dx$ | $-\frac{\cos 5x}{5}+c$ |
| $\int \cos 5x dx$ | $\frac{\sin 5x}{5}+c$ |
| $\int e^{5x}dx$ | $\frac{e^{5x}}{5}+c$ |
| $\int \sin(x+5)dx$ | $-\cos(x+5)+c$ |
| $\int \cos(x+5)dx$ | $\sin(x+5)+c$ |
| $\int e^{x+5}dx$ | $e^{x+5}+c$ |
| $\int (x+5)^3 dx$ | $\frac{(x+5)^4}{4}+c$ |
| $\int \sin(1-x)xdx$ | $\cos(1-x)+c$ |
| $\int \cos(1-x)dx$ | $-\sin(1-x)+c$ |
| $\int e^{1-x}dx$ | $-e^{1-x}+c$ |
| $\int (1-x)^3 dx$ | $-\frac{(1-x)^4}{4}+c$ |

**4.**

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|---|---|
| $\int \frac{1}{5x+3}dx$ | $\frac{\ln(5x+3)}{5}+c$ |
| $\int \frac{1}{3-5x}dx$ | $\frac{\ln(3-5x)}{-5}+c$ |
| $\int \frac{1}{x-3}dx$ | $\ln(x-3)+c$ |
| $\int \frac{1}{3-x}dx$ | $-\ln(3-x)+c$ |
| $\int \frac{8}{2x+1}dx$ | $4\ln(2x+1)+c$ |
| $\int \frac{8}{1-2x}dx$ | $-4\ln(1-2x)+c$ |
| $\int \frac{a}{bx+c}dx$ | $\frac{a}{b}\ln(bx+c)+k$ |

**5.**

| | |
|---|---|
| $\int \frac{1}{(5x+3)^2}dx$ | $-\frac{1}{5}\frac{1}{(5x+3)}+c=-\frac{1}{25x+15}+c$ |
| $\int \frac{1}{(3-5x)^2}dx$ | $\frac{1}{5}\frac{1}{(3-5x)}+c=\frac{1}{15-25x}+c$ |
| $\int \frac{1}{(x-3)^2}dx$ | $-\frac{1}{x-3}+c=\frac{1}{3-x}+c$ |
| $\int \frac{1}{(3-x)^2}dx$ | $\frac{1}{3-x}+c$ (it is the same as above) |
| $\int \frac{8}{(2x+1)^2}dx$ | $-\frac{4}{2x+1}+c$ |
| $\int \frac{8}{(1-2x)^2}dx$ | $\frac{4}{1-2x}+c$ |
| $\int \frac{a}{(bx+c)^2}dx$ | $-\frac{a}{b}\frac{1}{bx+c}+c$ |

**6.** $f(x)=x^{\frac{3}{2}}$

(a) $f'(x)=\frac{3}{2}x^{\frac{3}{2}-1}=\frac{3}{2}x^{\frac{1}{2}}$ (or $\frac{3}{2}\sqrt{x}$)

(b) $\int x^{\frac{3}{2}}\mathrm{d}x=\frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1}+c=\frac{2}{5}x^{\frac{5}{2}}+c$ (or $\frac{2}{5}\sqrt{x^5}+c$)

**7.** (a) $\int\sin(3x+7)\mathrm{d}x=-\frac{1}{3}\cos(3x+7)+C$

(b) $\int e^{-4x}\mathrm{d}x=-\frac{1}{4}e^{-4x}+C$

(c) $\int\cos(1-x)\mathrm{d}x=-\sin(1-x)+C$

**8.** (a) $f'(x)=(2\cos(5x–3))\,5\ (=10\cos(5x-3))$
$f''(x)=–(10\sin(5x–3))\,5=–50\sin(5x-3)$

(b) $\int f(x)\mathrm{d}x=\frac{2}{5}\cos(5x-3)+c$

**9.** (i) $f'(x)=5(3x+4)^4\times 3\ \left(=15(3x+4)^4\right)$

(ii) $\int(3x+4)^5\mathrm{d}x=\frac{1}{3}\times\frac{1}{6}(3x+4)^6+c\left(=\frac{(3x+4)^6}{18}+c\right)$

**10.** (i) $f'(x)=3(2x+5)^2\times 2=6(2x+5)^2$

(ii) $\int f(x)\mathrm{d}x=\frac{(2x+5)^4}{4\times 2}+c$

**11.** $f'(x)=\cos x\Rightarrow f(x)=\sin x+C$

$f\left(\frac{\pi}{2}\right)=-2\Rightarrow -2=\sin\left(\frac{\pi}{2}\right)+C\Rightarrow\ C=-3$

$f(x)=\sin x-3$

**12.** $y = 5e^{2x} - 5x + C$

substituting (0, 8): $8 = 5 + C \Rightarrow C = 3$

$y = 5e^{2x} - 5x + 3$

substituting $x = 1$

$y = 5e^2 - 2$ (= 34.9 )

**13.** $f(x) = \int \left( \frac{1}{x+1} - 0.5\sin x \right) dx = \ln(x + 1) + 0.5\cos x + c$

$2 = \ln 1 + 0.5 + c \Rightarrow c = 1.5$

$f(x) = \ln(x + 1) + 0.5\cos x + 1.5$

**14.** $f(x) = -\frac{1}{2}e^{-2x} - \ln(1 - x) + c$

Substituting : $4 = -\frac{1}{2} - \ln 1 + c \Rightarrow c = 4.5$

$f(x) = -\frac{1}{2}e^{-2x} - \ln(1 - x) + 4.5$

**15.** $f(x) = \int \sin(2x - 3)dx = -\frac{1}{2}\cos(2x - 3) + C$

initial condition: $4 = -\frac{1}{2}\cos 0 + C \Rightarrow C = 4.5$

$f(x) = -\frac{1}{2}\cos(2x - 3) + 4.5$

**16.** (a) $\int \frac{1}{2x+3} dx = \frac{1}{2} \text{In}(2x + 3) + C$

(b) $\int \frac{1}{(2x+3)^2} dx = \int (2x+3)^{-2} dx = \frac{1}{2} \frac{(2x+3)^{-1}}{-1} + C = -\frac{1}{4x+6} + C$

(c) $\int \frac{1}{(2x+3)^3} dx = \int (2x+3)^{-3} dx = \frac{1}{2} \frac{(2x+3)^{-2}}{-2} + C = -\frac{1}{4(2x+3)^2} + C$

## SUBSTITUTION

**17.** (i) $\int \frac{6}{2x-3} dx = 3\ln(2x - 3) + c$

(ii) $u = 2x - 3 \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$. Thus $\int \frac{6}{u} \frac{du}{2} = 3\int \frac{1}{u} du = 3\ln u + c = 3\ln(2x - 3) + c$

**18.** $\int \frac{6x}{2x^2 - 3} dx$. $u = 2x^2 - 3 \Rightarrow \frac{du}{dx} = 4x \Rightarrow dx = \frac{du}{4x}$

Thus $\int \frac{6x}{u} \frac{du}{4x} = \frac{3}{2} \int \frac{1}{u} du = \frac{3}{2} \ln u + c = \frac{3}{2} \ln(2x^2 - 3) + c$

**In the following questions only the final answers are given and the corresponding substitutions**

**19.** (i) $\int 6e^{3x+7}dx = 2e^{3x+7}$ $\quad (u = 3x+7)$

(ii) $\int 6xe^{3x^2+7}dx = e^{3x^2+7}$ $\quad (u = 3x^2+7)$

(iii) $\int 6x^2e^{3x^3+7}dx = \frac{2}{3}e^{3x^3+7}$ $\quad (u = 3x^3+7)$

**20.** (i) $\int 3x\sin(x^2+1)dx = -\frac{3}{2}\cos(x^2+1)$ $\quad (u = x^2+1)$

(ii) $\int 3x^2\cos(x^3+1)dx = \sin(x^3+1)$ $\quad (u = x^3+1)$

**21.** $\int \frac{3x^2}{x^3+1}dx = \ln(x^3+1)$ $\quad (u = x^3+1)$

**22.** $\int \frac{(\ln x)^5}{x}dx = \frac{(\ln x)^6}{6}+c$ $\quad (u = \ln x)$

**23.** $\int \frac{\tan^5 x}{\cos^2 x}dx = \frac{\tan^6 x}{6}+c$ $\quad (u = \tan x)$

**24.**

| | |
|---|---|
| $\int_0^1 (e^x+2)dx$ | = e+1 |
| $\int_0^{\pi} (\sin x+\cos x)dx$ | =2 |
| $\int_1^e \frac{7}{x}dx$ | = 7 |
| $\int_0^1 e^{2x+3}dx$ | $\frac{e^5-e^3}{2}$ |
| $\int_0^4 \frac{1}{x+1}dx$ | = ln5 |

**25.** (a) $f'(x) = \ln x$ (b) 3ln3-2

**26.** $\int \frac{1}{2x+3}\text{dx} = \frac{1}{2}\text{In}(2x+3)+C$

$\int_0^3 \frac{1}{2x+3}\text{dx} = \left[\frac{1}{2}\ln(2x+3)\right]_0^3 = \frac{1}{2}\ln 9 - \frac{1}{2}\ln 3 = \frac{1}{2}\ln 3 = \ln\sqrt{3}$

Thus $P = 3$

**27.** $\int_3^k \frac{1}{x-2}\text{dx} = [\ln(x-2)]_3^k = \ln(k-2) - \ln 1$

ln (k – 2) – ln1 = ln 7 $\Rightarrow$ $k-2=7$ , thus k = 9

**28.**

| | |
|---|---|
| $\int_7^5 \frac{f(x)}{4} dx$ | $-2$ |
| $\int_5^8 f(x)dx - \int_7^8 f(x)dx$ | 8 |
| $3\int_5^6 f(x)dx + \int_6^7 3f(x)dx$ | 24 |
| $\int_8^{10} f(x-3)dx$ | 8 |
| $\int_{2.5}^{3.5} f(2x)dx$ | 4 |

**29.** (a) $\int_c^d f(x-2)\,\mathrm{d}x = 8$ , $c = 2, d = 5$

(b) $\int_a^b f(2x)dx = 4$, $a = 0, b = 1.5$

(c) $\int_0^3 \left(f(x) + e^x\right)dx = \int_0^3 f(x)dx \int_0^3 e^x dx = 8 + e^3 - 1 = 7 + e^3$

## B. Paper 2 questions (LONG)

**30.** (i) $\int 2\sqrt{x+3}dx = \frac{4}{3}(x+3)^{3/2}$ $(u = x+3)$

(ii) $\int 2x\sqrt{x^2+3}dx = \frac{2}{3}(x^2+3)^{3/2}$ $(u = x^2+3)$

(iii) $\int 2x^2\sqrt{x^3+3}dx = \frac{4}{9}(x^3+3)^{3/2}$ $(u = x^3+3)$

**31.** $A = \int \cos x e^{\sin x} dx = e^{\sin x}$ $(u = \sin x)$

$B = \int \cos x\sqrt{\sin x}dx = \frac{2}{3}(\sin x)^{3/2}$ $(u = \sin x)$

$C = \int \sin^2 x \cos x dx = \frac{\sin^3 x}{3} + c$ $(u = \sin x)$

$D = \int \cos^3 x \sin x dx = -\frac{\cos^3 x}{3} + c$ $(u = \cos x)$

**32.** (i) $\int \frac{2x + \cos x}{x^2 + \sin x} dx = \ln(x^2 + \sin x)$ $(u = x^2 + \sin x)$

(ii) $\int e^x\sqrt{e^x+1}dx = \frac{2}{3}(e^x+1)^{3/2}$ $(u = e^x + 1)$

(iii) $\int (3x^2 + \cos x)(x^3 + \sin x)^3 dx = \frac{(x^3 + \sin x)^4}{4} + c$ $(u = x^3 + \sin x)$

**33.** (i) $\int \frac{(\ln x)^2}{x} dx = \frac{(\ln x)^3}{3} + c$ (ii) $\int \frac{\sqrt{\ln x}}{x} dx = \frac{2}{3}(\ln x)^{\frac{3}{2}} + c$

(iii) $\int \frac{1}{x(\ln x)^2} dx = -(\ln x)^{-1} + c = -\frac{1}{\ln x} + c$ (iv) $\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} + c$